Trio by the Golden Ratio Tuning for Viola, Cello and Contrabass

Notation for performance

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In this work, the tuning pitches of strings are calculated by Golden Ratio. Golden ratio is a number, 1.618 (approximately 5 of 8th), which appears often in nature, art, architecture and so on. The pitches are calculated by $f_{n+1} = G*f_n$ (here G = 1.618) repeatedly (Fig.1). When the multiplied f_{n+1} becomes over twice of initial pitch f_0, the f_{n+1} is divided by 2. Using this method, the obtained scale by golden ratio is shown in Fig 1.

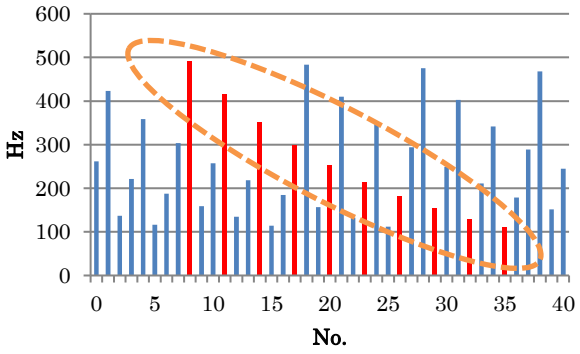

Fig.1 Pitch pattern calculated by Golden ratio and selection for scale

The tuning pitches for each instrument by the golden ratio based on A = 442 Hz are as follows,

Viola - I : 442 Hz (A, normal), II: 307.8 Hz(near D#), III: 208.2 Hz (near G#), IV: 122.9 Hz(near H)

Cello - I : 221 Hz (A, normal), II: 153.9 Hz(near D#), III: 90.8 Hz (near F#), IV: 63.3 Hz(between H-C)

Contrabass - I : 90.8 Hz(near F#) , II: 65.2 Hz (near C), III: 55.5 Hz(A, normal), IV: 38.5Hz (near D#)

Players can tune these pitches using application software in smart phone.

The structure of this piece is also based on the golden ratio. The golden ratio is the limit value obtained by dividing consecutive numbers in the Fibonacci sequence. Fibonacci sequence is as follows,

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610,… .

Here, the count in a bar is constructed by Fibonacci number as shown in Fig. 2.

Total 610 counts										
377							233			
1st mov. 144			2nd mov. 89		3rd mov.	4th mov.	5th mov.	6th mov.	7th mov.	8th mov.
55	34	55	34	55	89	35	55	34	55	89

Figure 2. Structure of music. The number of counts in each movement is based on Fibonacci num.

The note value and pitch are also based on Golden ratio. The motifs in this piece (note value and interval) are calculated by computer program (Typical examples are 2nd mov. And 3rd mov.). The computer program generates the note pitch and note value using Mandelbrot mapping repeatedly.

But, the articulation and dynamic mark etc. are added by composer in order to the more artistic expression (e.g. 1st mov.). This means that this music is not merely computer made music.

The other special notes in this work are following,

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: I, II, III, IV denote strings. III is D string in violin.
Put finger on written note and play natural harmonics,

-  : swing bowing, with a little fast bowing,

- vib. : harmonics tone with vibrato,

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accelerando, ad lib., non digitally